Abbott Lawrence Academy 2019-2020 Curriculum Map:
Year at a Glance
Subject: Advanced Honors Precalculus Grade: 11th

Unit Title	Time Allocation (# weeks based on 38 weeks in school year)	Essential Questions (for unit)	Core Text/Supplemental Learnings (include major references)	Performance Tasks (How will you know that students have mastered the taught concepts)
1. Analyzing Trigonometric Functions	**5 weeks**	• **Where are the turning points of the cosine and sine functions?** • **What is a radian?** • **How can you use a graph of $y = sin\ x$ to estimate solutions to the equation $sin\ x = -0.6$?** • **How are the six trigonometric functions defined?** • **Why does the sin^{-1}function on a calculator only return results between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$?** • **How many solutions are there to the equation such as $cos\ x = 0.8$?** • **Given the maximum and minimum values of a cosine or sine function, how do you find the amplitude and vertical displacement?** • **How can you make a sinusoidal function that has a specific period?** • **How can use a sinusoidal function to model periodic phenomena?**	**CME Project: Precalculus Common Core** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo: Virtual Classroom** **SMART Labs** **Khan Academy**	**SWBAT answer the following types of questions:** • **How can you use a graph of $y = sin\ x$ to estimate solutions to the equation $sin\ x = -0.6$?** • **Why does the sin^{-1}function on a calculator only return results between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$?** • **How many solutions are there to the equation such as $cos\ x = 0.8$?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how:** • **The relationship between degree and radian measure as the length of an arc on the unit circle subtended by a central angle** • **Relate the motion of an object around a circle to the graphs of the cosine, sine, and tangent functions** • **Solve equations that involve cosine and sine** • **The several relationships between the tangent function and the unit circle** • **Sketch and describe the graph of the tangent function** • **Define an inverse of a cosine, sine, and tangent** • **Recognize three other trigonometric functions: secant, cosecant, and cotangent** • **Make sense of sinusoidal functions in the context of real-world application** • **Understand the geometry of sinusoidal functions** • **Model with sinusoidal functions** **Students are responsible for a unit project that relies on using the learned skills to express the relationship of "Trigonometry " of the unit square by exploring how one might define trigonometry on the unit square as opposed to the unit circle. They will build a construction of the square trigonometry to explain what happens and justify answers.** **Assessments: Tests, quizzes, homework, classwork, video project**

2. Complex Numbers and Trigonometry	**4 weeks**	**● How can you write a complex number using trigonometry? ● What are the magnitude and argument of a complex number, and how do you find them? ● How do you use geometry to calculate something like $\left(1-i\sqrt{3}\right)\left(-3\sqrt{3}+3i\right)$? ● How can you test to see if an equation might be an identity? ● How can you use complex numbers to find formulas for $\cos 2x$ and $\sin 2x$? ● How can you use identities to prove other identities? ● How do you use De Moivre's Theorem to write a rule for an equation like $\cos 3x$? ● How can you connect roots of unity to regular polygons? ● For what values of $\cos x$ does $\cos 3x = 0$?**	**CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy Research:**	**SWBAT answer the following types of questions: ● How do you use geometry to calculate complex numbers? ● How can you use complex numbers to find formulas for cos 2x and sin 2x? ● For what value of cos x does cos 3x= 0? ● How do you use De Moivre's Theorem to write a rule for a trigonometric function? Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to: ● Graph complex numbers in the complex plan to use geometry to explain arithmetic fact of complex numbers ● Represent complex numbers using both rectangular coordinates and polar coordinates ● Test trigonometric functions to predict identities and show the Multiplication Law for complex numbers ● Use Pythagorean identities and algebra to prove the trigonometric equations is an identity ● Calculate powers of complex numbers using De Moivre's Theorem ● The geometry of roots of unity and the connections to roots of equations in a certain form. ● Find exact algebraic expression for certain trigonometric values Students are responsible for a unit project that relies on using the learned skills to read through a mathematical proof and make it their own to understand the reasons behind the steps. Students will use develop a formula for determining roots of a general cubic. Students will be able to understand how complex numbers crop in their development Assessments: Tests, quizzes, homework, classwork, video project**

3. Analysis of Functions	5 weeks	• How can you graph a polynomial function given its factored form? • How can you determine a polynomial's behavior at very large or very small inputs? • How can you use long division to find equations of secant or tangent lines to the graphs of a polynomial function? • What happens to equation like if $f(x) = \frac{3x^2+2x-1}{5x^2-3x+10}$ as x gets larger and larger? • Why do graphs of functions like $g(x) = \frac{x^2-15}{x-4}$ and $h(x) = \frac{x^2-16}{x-4}$ look so different from each other? • How can you find tangent lines to rational functions? • What happens when interest is compounded more and more frequently? • What are some reasons to introduce the number e? • How can you relate any exponential or logarithmic function to like $f(x) = e^x$ and $g(x) = \ln x$?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses on analysis of functions which students will do by thinking about continuity and the slope of a line tangent to a graph at a point, students will part take in the following: • In-class discussions: generalize the process and find an equation for the slope of the tangent to the graph of an exponential or logarithm function by finding equations for the tangent to the graph of various polynomial functions and rational functions. • Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ average rate of change ○ continuous ○ continuously compounded interest ○ determinant ○ e ○ hole ○ infinite discontinuity ○ instantaneous speed ○ linear fractional transformation, R_A ○ natural logarithm, ln x ○ power function ○ removable discontinuity ○ secant line ○ structure-preserving map ○ tangent line • Pictorial representations: students will be to visualize a line through those two points—a secant line by relating the graph of a function to the slope of its tangent at any point and use that information to analyze the graph. • Project: Partial Fractions: students will demonstrate how to use the partial fraction decomposition of a rational function to evaluate sums and to graph rational function. Students will be able to explain how to produce a partial fraction decomposition for certain rational functions. Other common forms of assessments include: test/quizzes/homework/classwork(calculations)
4. Combinatorics	4 weeks	• How many five-digit number can you make using only the digits 1 and 2? • In a kindergarten class, each student has four pictures: a square, a triangle, a circle, and a star. Each of the kids will make a design by gluing these four pictures in a line. There are 20 kids in the class. Can each child make a different design or will there have to be repeats? • What does it mean for two problems to be isomorphic? • In how many ways can you pick three objects, in order, from a set of six distinct objects? • How many three-digit numbers are there that have repeated digits? • How many five-student committees are possible in a class of 26 students? • What is the coefficient of the $x^{13}y^{37}$ term in the expansion of $(x+y)^{50}$?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses on making sense of the linear algebra focusing on topics like solving systems of equations in matrix format. Students will learn use Gaussian Elimination. students will part take in the following: • In-class discussions defining how to use various methods that can be used to count the number of elements in a set without enumerating them. • Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ anagram ○ combination ○ isomorphic ○ permutation ○ ${}_pC_k$, number of combinations of n objects. Taken at k at a time ○ ${}_nP_k$, number of permutations of n objects, taken k at a time • Pictorial representations: students will create various graphing techniques and tables to use the number of counting strategies to develop the habit of looking at the mathematical structure to determine what strategies make sense.

		● What is the connection between the Pascal's Paths problem and the entries in Pascal's Triangle? ● What is the sum of the entries in row n of Pascal's Triangle 2^n?		● Project: The Simple Locks: students will be presented with certain rules to find the "combination" to the simplex lock by using the correct permutation from all of the possible codes.9 Other common forms of assessments include: test/quizzes/homework/classwork(calculations)
5. Functions and Tables	5 weeks	● What are the differences between a closed-form definition and a recursive definition for a function? ● How can you prove that a closed-form and a recursive function definition agree at each of infinitely many inputs? ● What happens to the ratio of consecutive Fibonacci numbers? ● If the third differences in the table of a polynomial function are all 24, what can you say about that function? ● What are Mahler polynomials? ● How can you use differences to find a polynomial function that fits a table? ● What methods are available for deciding if a linear, polynomial, or exponential rule fits a table? ● How can you find a closed-form function definition that satisfies a two-term recurrence? ● What is the monthly payment for a three-year car loan for $15,000, taken out at 5% APR?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses connections to the properties of Pascal's Triangle with properties of difference tables. Students will part take in the following: ● In-class discussions: students will talk how to connect the properties of Pascal's Triangle with properties of difference table and use the connections to construct and verify functions that fit tables. ● Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ base case ○ closed-form definition ○ difference table ○ equilibrium point ○ Fibonacci sequence ○ functional equation ○ hockey-stick property ○ Mahler polynomials ○ mathematical induction ○ recurrence ○ recursive definition ○ two-term recurrence ○ up-and-over property ● Pictorial representations: students will model and work with closed-form and recursive definitions of functions by using induction to prove that the functions agree. ● Project: Rhyme Schemes: students will explore how to find quick ways to determine the number of different types of rhyme schemes that are possible given poems of different numbers of lines. Other common forms of assessments include: test/quizzes/homework/classwork(calculations)
6. Analytic Geometry	5 weeks	● What is the set of points equidistant from the x-axis and the point (0,4)? ● How can you use coordinates to show that the diagonals of a parallelogram bisect each other? ● How can you find the center and radius of a circle with an equation written in normal form? ● How do you slice an infinite double cone with a plane to get a parabola? ● What is the locus definition of a hyperbola? ● What kind of conic section do you get when you graph $x^2 + 16y^2 - 8x + 64y + 64 = 0$? How can you identify the conic section from its equation? ● How can you interpret the matrix operations of sum and scalar product geometrically?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses on learning that by choosing the "generic" coordinates students can simplify their algebraic calculations and see more meaning in them. Students will part take in the following: ● In-class discussions defining how a slicing plane intersects a double cone at different angles, cutting off different conic sections. ● Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ affine combination ○ centroid ○ conic sections ○ convex combination ○ coordinatize ○ Dandelin sphere ○ directix ○ double cone, $z^2 = x^2 + y^2$

		• Why might it be useful to write an equation of a line in vector form? • how can you use vectors to prove that the medians of a triangle are concurrent?	Khan Academy	○ eccentricity ○ ellipse ○ focus, foci ○ head and tail of a vector ○ hyperbola ○ locus ○ major axis ○ minor axis ○ parabola ○ parameter ○ point-tester ○ power of a point, $\prod_{.}(P)$ ○ signed power of a point, $\prod_{s}(P)$ ○ vector, $\overrightarrow{AB}$ • Pictorial representations: students will create visually create vectors on a coordinate plane and they will model conic sections as wheel as coordinate geometry. • Project: The General Quadratic: students will analyze the graph of the general quadratic. Other common forms of assessments include: test/quizzes/homework/classwork(calculations)
7. Probability and Statistics	6 weeks	• If you are to roll four number cubes, what is the probability they sum to 12? • What is expected value? • How can you use polynomials to solve probability problems? • How can you calculate the standard deviation for a large set of data? • What happens to the mean, variance, and standard deviation if an experiment is repeated a second time? • What is the mean and standard deviation for the number of heads on 400-coin flips? • What is the Central Limit Theorem? • Why is the normal distribution so common? • What is the probability of rolling 10% or fewer sizes if you roll 1000 number cubes?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses on probability and statistical analysis by using patterns in the form of calculation to draw out conclusions with having to do complex calculations. Students will part take in the following: • In-class discussions will revolve around having students explain how they have connected the result of the calculations in context with the problem to draw conclusions about probability situations. • Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ Bernoulli trial ○ conditional probability ○ confidence interval ○ control group ○ cumulative density function ○ event ○ expected value, E(x) ○ experiment ○ experimental probability ○ frequency, $\|A\|$ ○ independent ○ mean absolute deviation ○ mean squared deviations, or variance, σ^2 ○ mutually exclusive ○ normal distribution, $N(\mu,\sigma)$ ○ observational study ○ population parameter ○ probability density function ○ probability histogram ○ random sampling ○ random variable

				○ root mean squared deviation, or standard deviation, σ ○ sample proportion ○ sample space ○ sample statistic ○ sample survey ○ theoretical probability ○ treatment group ○ z-score ● Pictorial representations: students will create probability distributions for a series of repeated independent trials of a probability experiment in several different contexts to similarities between the graphs to understand the Central Limit Theorem. ● Project: Faking the Flips: students will be asked to make “truly” random data, and try to make a believable fake. Other common forms of assessments include: test/quizzes/homework/classwork(calculations)
8. Ideas of Calculus	4 weeks	● How can you find the area of an irregularly-shaped figure? ● How can you estimate the area under a curve? ● What is the area of the region under the graph of $y = x^2$ from $x = 0$ to $x = 1$? ● What is a closed-form expression for the function defined by $F(b) = s(1,b)(x^3)$, where $b > 1$? ● What is a closed-form expression from the function defined by $F(b) = s(1,b)(x^4)$, where $b > 1$? ● What is Fermat’s approach to finding the area under the graph of $y = x^m$ between $x = 0$ and $x = 1$? ● What is the value of $S[1,2](x^{-2})$? ● What is the value of $S[1,2](x^{-1})$? ● What is the value of $S[1,2](e^x)$?	CME Project: Precalculus Common Core https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ Edmodo: Virtual Classroom SMART Labs Khan Academy	Students will work in a variety of way to demonstrate mastery of the content. This unit focuses on the introduction of the ideas that underlie calculus and several different ways of thinking. Students will Students will part take in the following: ● In-class discussions defining how to find the areas of shapes, areas under curves, and how a function emerges. ● Mathematical reflections: Students will be assigned mathematical reflections as a study guide to make sure that students comprehend key concepts like: ○ Fermat lower sum, $LF_n(a \cdot b)(f(x))$ ○ Fermat upper sum, $UF_n(a,b)(f(x))$ ○ lower sum, $L_n(a,b)(f(x))$ ○ upper sum, $U_n(a,b)(f(x))$ ○ $S(a,b)(f(x))$ ● Pictorial representations: students will create and use a variety of tools to find area from graphing to formulas. ● Project: A Delightful Sequence of Polynomials: students will learn about sequences of polynomials called the Chebyshev polynomials and the applications of polynomials to produce multiple angle formulas for cosine. Other common forms of assessments include: test/quizzes/homework/classwork(calculations)

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 1 “Analyzing Trigonometric Functions” (5) Weeks

Essential Questions	• **Where are the turning points of the cosine and sine functions?** • **What is a radian?** • **How are the six trigonometric functions defined?** • **Given the maximum and minimum values of a cosine or sine function, how do you find the amplitude and vertical displacement?** • **How can you make a sinusoidal function that has a specific period?** • **How can use a sinusoidal function to model periodic phenomena?**
Learning Objectives for Unit	**Experimentation** … Students will learn to… • (Experimentation) work with constructions to visualize trigonometric functions. They will learn to answer questions about what they observe by making conjectures and proving theorems that they might have made conjectures about. • (Encapsulation) develop the ability to recognize regularities in a process to analyze the steps of the process building an algorithm. Students will be able to recognize functions and transform basic trigonometric graphs.
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** • **How can you use a graph of** $y = sin\ x$ **to estimate solutions to the equation** $sin\ x = -0.6$**?** • **Why does the** sin^{-1}**function on a calculator only return results between** $-\frac{\pi}{2}$ **and** $\frac{\pi}{2}$**?** • **How many solutions are there to the equation such as** $cos\ x = 0.8$**?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how:** • **The relationship between degree and radian measure as the length of an arc on the unit circle subtended by a central angle** • **Relate the motion of an object around a circle to the graphs of the cosine, sine, and tangent functions** • **Solve equations that involve cosine and sine** • **The several relationships between the tangent function and the unit circle** • **Sketch and describe the graph of the tangent function** • **Define an inverse of a cosine, sine, and tangent** • **Recognize three other trigonometric functions: secant, cosecant, and cotangent** • **Make sense of sinusoidal functions in the context of real-world application** • **Understand the geometry of sinusoidal functions** • **Model with sinusoidal functions** **Students are responsible for a unit project that relies on using the learned skills to express the relationship of “Trigonometry “ of the unit square by exploring how one might define trigonometry on the unit square as opposed to the unit circle. They will build a construction of the square trigonometry to explain what happens and justify answers.** **Assessments: Tests, quizzes, homework, classwork, Video project**

CC Standards/ Lawrence Standards	**Language Objectives** The reading, speaking, writing, and listening skills will you teach,	**Academic Language** The formal-language skills- vocabulary,	**Content Objectives** What students will know and be able to do at the end of the unit	**Texts and Supplemental Learnings**	**Cross-Content Connections**

	re-teach, or review so students will be able to explain and apply the content, skills, and/or procedures.	grammar, punctuation, syntax, discipline-specific terminology, or rhetorical conventions—that allow students to acquire knowledge			
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A.REI.4** Solve quadratic equation in one variable. **A.SSE.1** Interpret expression that represent a quantity in terms of its context **A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients. **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum and	Students will be able to: ● convert degree measurements to radian ● explain how the radian measure is used as a distance measurement ● graph sine and cosine functions and define a periodic function ● solve equations that involve sine and cosine ● list the trigonometric functions by learning the tangent and the three reciprocal functions: secant, cosecant, and cotangent ● model graphs of the above functions and demonstrate each function in the unit circle to work with the basic identities ● define one-to-one functions and inverse functions ● how to restrict the domain of cosine, sine, and tangent in order to define the inverses of these functions ● emphasize and validate the idea that trigonometric functions are treated as "functions as usual" ● discuss the sinusoidal function transformations through amplitude and shift changes	● amplitude ● arc ● asymptote ● central angle ● decreasing ● increasing ● inverse function ● maximum ● minimum ● period ● periodic ● phase shift ● Pythagorean Identity ● radian ● secant line ● sinusoidal function ● turning point ● vertical displacement	*Learning Goals:* SWBAT- ● discuss the relationship between degree and radian measure as the length of an arc on the unit circle subtended by a central angle ● relate the motion of an object around a circle to the graphs of the cosine and sine functions ● solve equations that involve cosine and sine (such as $3\cos\ x\ + 2 = 1$) ● compare and contrast several relationships between the tangent function and the unit circle ● sketch and describe the graph of the tangent function ● define an inverse of cosine, sine, and tangent ● recognize and define three other trigonometric functions: secant, cosecant, and cotangent ● explain how they made sense of sinusoidal functions in the context of previous experience ● model the geometry of sinusoidal functions ● model with sinusoidal functions *Habits and Skills:* SWBAT- ● calculate cosine and sine using radians directly without converting to degree ● visualize periodic functions, and identify their period	**CME Project: Precalculus Common Core-Chapter 1 (pages 2-81)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo: Virtual Classroom** **Khan Academy** https://www.khanacademy.org/math/algebra2/trig-functions/constructing-sinusoids-alg2/v/trig-function-equation https://www.khanacademy.org/math/algebra2/trig-functions/amplitude-and-midline-of-sinusoids-from-formulas-alg2/v/we-amplitude-and-period **Other:** http://tutorial.math.lamar.edu/pdf/Trig_Cheat_Sheet_Reduced.pdf http://www.epcc.edu/OfficeofStudentSuccess/tutorialservices/tutorialsupportservicesMDP/Documents/Trigonometry%20Handouts%20(PDF)/Math%20Handout%20(Trigonometry)%20Trig%20Formulas%20Web%20Page.pdf **Glencoe Common Core Algebra II text** **Glencoe Common Core Precalculus text**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Geometry:** The students will learn to graph the unit circle to work with trigonometric function to prove the Pythagorean Identity. **Physics:** The students will learn to graph functions like sine and cosine to define periodic functions and how a period is defined. **SAT Prep:** The students will learn to think about standardized test questions like arc length relate it to the central angle for related proportional rates problem. **Biology:** Students learn about the behaviors of a function in a manner that relates to biological functions like a period function. **Algebra II:** Student will continue to build on the parent functions that were learned from Algebra to build on concepts relating to more complicated analysis of the unit circle. **Trigonometry**: Students will build on the concepts of the unit circle and the trigonometric functions and how they relate to concepts of Precalculus. **Chemistry:**

minimum value of the function it defines. **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F.IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F.BF.1.a**			• demonstrate how to "undo" cosine or sine to solve equations • compare the cosine and sine functions through their relation to the unit circle and through their graphs • model the unit circle to generate the graph of $y = tan\ x$ • visualize geometrically the tangent and secant functions by graphing • restrict the domain of a function to make it one-to-one • solve equations and prove identities using trigonometric functions • demonstrate and show the graph of $y = Asin\ \left(ax + b\right) + B$ is simply a transformation of the basic graph of $y = sin\ x$		Students are starting to see behaviors of functions as a method of analyzing the transformation and shift changes similar to what students in labs. Students are learning to validate ideas and concepts to understand their hypothesis. **History:** Students are building concepts of relationship to better understand and interpret key behaviors similar to how they discuss and emphasize historical events with plausible causations and effects. **English/Speech:** Students are expected to relate concepts to build mathematical arguments to prove or disprove behaviors similar to how they would write persuasive essays or develop a thesis to prove or disprove their statements or those others.

Determine an explicit expression, a recursive process, or steps for calculations from a context **F.BF.1.c** (+) Compose functions. **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms **F.BF.3** Identify the effect on the graph of replacing *f(x)=* by *f(x)+k, k f(x), f(kx),* and *f(x +k)* for specific values of k (both positive and negative values); find the value of *k* given the graphs. Experiment with the cases and illustrate an explanation of the effects on the graph using technology. **F.BF.4** Find inverse functions. **F.BF.4.a** Solve an equation of the form f(x)=c for a simple function f that has an inverse and write an expression for the inverse. **F.BF.4.b** (=) Verify by composition that one function is the inverse of another. **F.BF.4.c** (=) Read values of an inverse function from as graph or a table, given that the function has an inverse. **F.BF.4.d** (+) Produce an invertible function from a non invertible function by restricting the domain. **F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.					

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric function to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F.TF.3** (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π+ x and 2π-x in terms of their values for x, where x is any real number. **F.TF.4** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **F.TF.6** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows it inverse to be constructed. **F.TF.7** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. **F.TF.8** Prove the Pythagorean $sin^2\theta + cos^2\ \theta\ = 1$ identity and use it to calculate trigonometric ratios.					

G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related					

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Subject: Advanced Honors Precalculus Grade: 11th
Unit 2 "Complex Numbers and Trigonometry" (4) Weeks

Essential Questions	● **How can you write a complex number using trigonometry?** ● **What are magnitude and argument of complex numbers and how do you find them?** ● **How can you test to see if an equation is an identity?** ● **How can you use identities to prove other identities?** ● **How do you calculate powers of complex numbers using De Moivre's Theorem?** ● **How do I calculate complex numbers?** ● **How can you connect roots of unit to regular polygons?**
Learning Objectives for Unit	Students will learn to ● (Visualization) use the complex plane to visualize the complex numbers and calculations with complex number geometrically. ● (Logical Reasoning) make connections to the complex number and calculations with trigonometric functions to generate and prove trigonometric identities. ● (Extension) work with the process of calculating roots of unity with polynomials.
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **How do you use geometry to calculate complex numbers?** ● **How can you use complex numbers to find formulas for cos 2x and sin 2x?** ● **For what value of cos x does cos 3x= 0?** ● **How do you use De Moivre's Theorem to write a rule for a trigonometric function?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **Graph complex numbers in the complex plan to use geometry to explain arithmetic fact of complex numbers** ● **Represent complex numbers using both rectangular coordinates and polar coordinates** ● **Test trigonometric functions to predict identities and show the Multiplication Law for complex numbers** ● **Use Pythagorean identities and algebra to prove the trigonometric equations is an identity** ● **Calculate powers of complex numbers using De Moivre's Theorem** ● **The geometry of roots of unity and the connections to roots of equations in a certain form.** ● **Find exact algebraic expression for certain trigonometric values** **Students are responsible for a unit project that relies on using the learned skills to read through a mathematical proof and make it their own to understand the reasons behind the steps. Students will use develop a formula for determining roots of a general cubic. Students will be able to understand how complex numbers crop in their development**

		Assessments: Tests, quizzes, homework, classwork, video project			
CC Standards/ Lawrence Standards	**Language Objectives**	**Academic Language**	**Content Objectives**	**Texts and Supplemental Learnings**	**Cross-Content Connections**
A.APR.4 Prove polynomial identities and use them to describe numerical relationship. **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **A.REI.4** Solve quadratic equations in one variable. **A.REI.4.a** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x-p)^2=q$ that has the same solutions. Derived the quadratic formula from this form. **A.REI.4.b** Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a+/- bi for real numbers a and b. **A.SSE.1** Interpret expressions that represent a quantity in terms of its context. **A.SSE.1.a** Interpret parts of an expressions, such as terms, factors, and coefficient. **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. **F.TF.8** Prove the Pythagorean $sin^2\theta + cos^2\ \theta\ = 1$ identity and use it to calculate trigonometric ratios. **F.TF.9**	Students will be able to: ● explain how to determine the magnitude and argument of any complex numbers, orally and in writing ● model complex numbers using both rectangular coordinates and polar coordinates ● choose and describe when its best to use either rectangular or polar coordinates to represent complex numbers ● display the basic addition rules for cosine and sine using the Multiplication Law for complex numbers in writing ● test and verify trigonometric equations to predict whether they are identities ● model Pythagorean identities and algebra to prove that a trigonometric equation is an identity ● explain how to calculate powers of complex numbers using De Moivre's Theorem, orally and in writing ● compare and contrast the geometry of roots of unity ● analyze the connection to roots of equations of the form x^n-1=0 ● calculate exact algebraic expression for certain trigonometric values ● solve roots of unity	● argument, arg (x) ● polar coordinates ● complex numbers ● conjugate, $\underline{z}$ ● cyclotomy ● discriminant ● identically equal ● identity ● magnitude, $\lvert z\rvert$ ● modulus ● norm, N(z) ● polar coordinates for complex numbers ● rectangular coordinates ● rectangular form for complex numbers ● roots of unity	*Learning Goals* SWBAT: ● Represent complex numbers using both rectangular coordinates and polar coordinates. ● Determine the magnitude and argument of any complex number. ● Decide when it is best to use wither rectangular or polar coordinates to represent complex numbers. ●Test trigonometric equations to predict whether they are identities. ● Show the basic addition rules for cosine and sine using the Multiplication Law for complex numbers. ● Use Pythagorean identities and algebra to prove that trigonometric equations is and identity. ● Calculate powers of complex numbers using De Moivre's Theorem. ● Understand the geometry of roots of unity, and the connection to roots of equations of the form x^n-1=0. ● Find exact algebraic expressions for certain trigonometric values. *Habits and Skills* SWBAT: ● Graph complex numbers in the complex plane.	**CME Project: Precalculus Common Core- Chapter 2 (pages 82-163)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **YouTube** https://www.youtube.com/watch?v=2lbABbfU6Zc **Other:** http://www.epcc.edu/OfficeofStudentSuccess/tutorialservices/tutorialsupportservicesMDP/Documents/Trigonometry%20Handouts%20(PDF)/Math%20Handout%20(Trigonometry)%20Trig%20Formulas%20Web%20Page.pdf	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Chemistry:** Students will learn to better understand roots by comparing how important these roots are to the behavior of the function which will allow students to practice predicting and analyzing how things work with proof. **Physics:** Students will reinforce topics like waves in this unit by thinking of the trigonometric functions in terms of waves and polar coordinates looking a vector to analyze the argument and magnitudes. **English:** Students will learn to imprete expression by looking at the context similar to the skills learned English and Speech and Composition to analyze data in this particular case to see parts as a single entity by focusing on one aspect. **History:** Students will learn how prove identities to use the relationship to better analyze and review the information similar to the skills learned in PreAP and AP World History. **Geometry:** Students will start to represent trigonometric functions on the unit circle and to learn how these coordinates can be related on the Argand plane using complex numbers which are similar to the polar coordinates that students will learn about in this unit

(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **N.CN.1** Know there is a complex number I such that $i^2 = -1$, and every complex number has the form a +bi with a and b real. **N.CN.2** Use the relation show $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N.CN.3** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **N.CN.4** (+) Represent complex number on the complex plane in rectangular and polar form, and explain why the rectangular and polar forms of a given complex number represent the same number. **N.CN.5** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. **N.CN.7** Solve quadratic equations with real coefficients that have complex solutions. **N.CN.8** (+) Extend polynomial identities to the complex numbers. **N.CN.9** (+) Know the Fundamental Theorem of Algebra; shoe that it is true for quadratic polynomials			● Use geometry to explain arithmetic fact of complex numbers ● Multiply two complex numbers in the form r(cos Ɵ + i sin Ɵ). ● Manipulate trigonometric expressions. Determine useful test cases and techniques to identify identities, ● Use basic rules to generate more complicated rules. ● Calculate with complex number. ● Visualize complex numbers and their arithmetic.		

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 3 "Analysis of Functions" (5) Weeks

Essential Questions	● **How can you graph a polynomial function given its factored form?** ● **How can you determine a polynomial's behavior at very large or very small inputs?** ● **How can you use long division to find equations of secant or tangent lines to the graph if a polynomial function?** ● **What are complex numbers?** ● **How can you use complex numbers to solve any quadratic equation?** ● **How do you represent a complex number graphically?** ● **What is the graphical effect of adding two complex numbers?** ● **How are the magnitude and argument of the product of two complex numbers related to the magnitude and argument of the original numbers?**
Learning Objectives for Unit	Students will learn to: ● (Reasoning by continuity) look at graphs of a function and chose two points on that graph to construct a secant line that will used to use the tangent and calculate slope. The slope of the constructed line is the limit of the slope of the secant line as the moving point approaches the fixed point. ● (Visualization) relate the graph of a function to the slope of its tangent at any point and use the information to analyze the graph. ● (Generalization) generalize the process and find an equation for the slope of the tangent to the graph of an exponential or logarithmic function without having a formal way to calculate the limit.
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **How can you graph a polynomial function given its factorial form?** ● **How can you determine a polynomial's behavior at a very large or very small inputs?** ● **How can you use long division to find equations of a secant or tangent lines to the graph of a polynomial function?** ● **What happens when interest is compounded more and more frequently?** ● **What are some reasons to introduce the number e?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **Develop both formal and informal understanding of what continuity means by using key theorems to draw conclusions about the properties of the function given the degree** ● **Explore rational functions to find equations for the slope of the line tangent to a rational function graph at a point.** ● **Analyze the exponential and logarithmic functions to define the continuously compounded interest and limit definition.** ● **Find the slopes of the lines secant or tangent to the graph of a polynomial function** ● **Learn that these slope represent average and instantaneous rates of change of the function** ● **Graph functions that re quotients of polynomials** ● **Pay attention to the values of x for which the denominator polynomial has a value of 0** ● **The constant e arises and different ways of computing it** ● **Find the natural log function and how to find the inverse of it** ● **Explain the effects of the discontinuities and the effects on graphs** ● **Graph rational functions using their horizontal and vertical asymptotes** **Students are responsible for a unit project that relies on using the learned skills to demonstrate the relationship how to use the partial fraction decomposition of a rational function to evaluate sum and to graph rational functions.** **Assessments: Tests, quizzes, homework, classwork, video project**

CC Standards/ Lawrence Standards	Language Objectives	Academic Language	Content Objectives	Texts and Supplemental Learnings	Cross-Content Connections
A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on the division by x-a is p(a), so p(a)=0 if and only if (x-a) is a factor of p(x) **A.APR.3** Identify zeros of polynomials when suitable factorization are available, and use the zeros to construct a rough graph of the function defined by the polynomial **A.APR.6** Rewrite simple rational expression in different forms; write a(x)/b(x) in the form (q(x)+r(x))/b(x), where polynomials are with the degree of r(x) less than the degree of b(x) using inspection, long division, or, for the more complicated examples, a computer algebra system **A.APR.7** Understand that rational expressions for a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expression. **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise **A.REI.11** Explain why the x-coordinates of the points where the graphs of the equation y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x) find the solutions approximately using technology to graph the functions, make tables of values, or find successive approximations. INclude cases where	● state the Change of Sign theorem and the intermediate Value Theorem for Polynomials ● analyze the graphs of polynomial functions ● find the equation of a line secant to a polynomial functions and the average rate of change of a function between two points ● write the Taylor expansion for a polynomial function about a point ● find the equation of the tangent to a polynomial curve at a point ● sketch the graph of a rational function including asymptotes and holes ● evaluate limits of rational expressions ● find the equation of the tangent to the graph of rational function at a point ● state and use the limit and factorial definitions of e and e^x ● use the inverse relationship between e^x and ln x to solve equations ● find the equation for the line tangent to the graph of y=e^x or y =ln x at a point	● average rate of change ● continuous ● continuously compounded interest ● determinant ● e ● hole ●infinite discontinuity ● instantaneous speed ● linear fractional transformation, R_A ● natural logarithm, ln x ●power function ● Removable discontinuity ● secant line ●structure-preserving map ● tangent line	*Learning Goals* SWBAT: ●state and use the limit definition of e and e^x ● state and use the factorial definition of e and e^x ● use the inverse relationship between e^x and ln x to solve problems ●find an equation for the tangent to the graph of y=e^x or y=ln x at a point ● sketch the graph of a rational function, including asymptotes and holes ●evaluate limits of rational expressions ● find the equation of the tangent to the graph of a rational function at a point ● use matrices to write linear fractional transformation of the function f(x)= 1/x ● state the Change of Sign Theorem and the Intermediate Value Theorem for Polynomials and use the to analyze the graphs of polynomial functions ● find the equations of a line a secant to a polynomial function and the average rate of change of a function between two points ● write the Taylor expansion for a Polynomial function bout a point ●find the equation of the tangent to a polynomial curve at a point *Habits and Skills* SWBAT: ● develop a definition of continuously compounded interest ● visualize relationships between the graphs of f(x) = e^x and g(x)= ln x and the slopes of the tangents to these graphs ● use functional equations to recognize the ln function as a logarithm	**CME Project: Precalculus Common Core-Chapter 3 (pages 164-273)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **You Tube** **Other:** http://www.epcc.edu/OfficeofStudentSuccess/tutorialservices/tutorialsupportservicesMDP/Documents/Trigonometry%20Handouts%20(PDF)/Math%20Handout%20(Trigonometry)%20Trig%20Formulas%20Web%20Page.pdf	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based from readings, lectures, collaborative work, assignments, etc. **Physics:** This particular chapter has the most connection to Physics especially since students will think about the relationship of factors and behaviors of graph to calculate the rate of change over time but focusing informally on the **Biology:** Students start to build on the concept of the rate of change by understand the concepts how the rate of change is affected between two points the student are building the relationship of the dependent and independent variables. **Algebra II:** Students will learn how to graph functions na study the behavior of the functions to evaluate limits of rational expressions to look at asymptotes and roots. **Geometry:** Students will think about how tangent functions work with connection to the rate of change given two points on a function. Students are learning how the behavior is limited by the two points used to measure the rate of change. **Trigonometry**: Students will view how concepts like the tangent and secant line can be used to measure the rate of change between two points to examine the relationship of the graph.

f(x) and/or g(x) are linear, polynomials, rational, absolute value, exponential, and logarithmic functions. **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression **F.BF.3** Identify the effect on the graph of replacing f(x) by f(x)+k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value if k given the graphs. Experiment with cases and illustrate an explanation of the effect on the graph using technology. **F.BF.4** Find inverse functions **F.BF.4.c** Read values of an inverse function from a graph or a table, given that the function has an inverse **F.BF.4.d** Produce an invertible function from a non invertible function by restricting the domain **F.IF.4** Fir a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship **F.IF.6** Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph **F.IF.7** Graph functions expressed symbolically and show key feature of the graph, by hand in simple cases and using technology for more complicated cases **F.IF.7.c** Graph polynomial functions, identifying zeros when suitable factorization are available and showing end behavior **F.IF.7.d** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior			● visualize different types of discontinuities, relating equations and their graphs ● reason logically to find limit at infinity ● find the equation of the tangent to the graph of a rational function ● visualize the graphs of a polynomial function from its factored form ● use the continuity of a polynomial function to draw conclusions about the function's behavior fat extreme values or about lines tangent to the graph of the function ● connect quotient in polynomial long division to Taylor expansions and to equations of tangent lines		

F.IF.7.e Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions showing period, midline, and amplitude **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function **F.LE.1.c** Compose functions **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents **N.VM.6** Use matrices to represent and manipulate data oi represent payoffs or incidence relationships in a network					

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 4 "Combinatorics" (4) Weeks

Essential Questions	• **What does it mean for two problems to be isomorphic?** • **How many ways can you pick object in order from a set of n elements?** • **How many combinations can be n ways given a certain number of n values?** • **What is the connection between the Pascal's Paths problem and the entries in Pascal's Triangle?** • **Why is the sum of the entries in row n of Pascal's Triangle 2^n?**
Learning Objectives for Unit	Students will learn to: • solve a simpler problem by aligning strategies to the recursive thinking •identify and create isomorphic problems • counting functions by counting the number of functions from one finite set to another • know when to use particular counting strategies
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** • **What is the coefficient $x^{13}y^{37}$term in the expansion of the $(x+y)^{50}$?** • **What is the difference between a combination and a permutation?** • **What is a isomorphic function?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:**

	- Techniques to solve counting problems that appear different by have the same underlying mathematical structure
	- Formal ways to approach counting problem, like what permutations and combinations are and how to count them
	- Count permutations (ordered choices) and combinations (unordered choices)
	- Count anagrams (ordered choices with repetitions)
	- The coefficients in the expansion of $(a+b)^n$ are
	    ○ The number in the nth row of Pascal's triangle
	    ○ The number of combinations when choosing from n elements
	
	Students are responsible for a unit project that relies on using the learned skills to use special cases of the covered material to develop only the properties of combination and permutation to think about how isomorphic functions work relating it to Pascal's mathematical discoveries.
	
	Assessments: Tests, quizzes, homework, classwork, video project

CC Standards/ Lawrence Standards	Language Objectives	Academic Language	Content Objectives	Texts and Supplemental Learnings	Cross-Content Connections
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle **Prepares for S.CP.9**	● develop strategies for systematic combinatorics ●interpret what combinatorics means can be used and explain what kinds of problems can be used for combinatorics ● develop and use formulas for the number of permutations of n objects taken at k times ● model the formula for combinations for the number of combinations at n things taken at k at a time ● find the number of anagrams for a given word ● apply the counting strategies to solve the Pascal's Path problems ● Explain why finding the coefficients of a binomial expansion of Pascal's Triangle ●Model and show the entries of Pascal's Triangle from a variety of prospective ●present combinatorial problems with formal strategies	● anagram ● combination ● isomorphic ● permutation ● ${}_pC_k$, number of combinations of n objects. Taken at k at a time ● ${}_nP_k$, number of permutations of n objects, taken k at a time	*Learning Goals* SWBAT: ● Apply counting strategies to solve the Pascal's Paths problem. ● Explain why the coefficients of a binomial expansion are found in Pascal's Triangle. ● See the entities of Pascal's Triangle from a variety of perspectives ● Develop and use formulas for finding the number of permutation of n objects, taken k at a time ● Find a formula for the number of combinations of n objects, taken k at a time ● Find the number of anagrams for a word ● Recognize the kids of problems that can solve using combinations ● Develop your own strategies for systematic counting *Habits and Skills* SWBAT: ● Use combinations to find binomial coefficients. ● Use combinatorics to probe the Binomial Theorem. ● Recognize isomorphic problems.	**CME Project: Precalculus Common Core-Chapter 4 (pages 274-339)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **Youtube**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Geometry/Trigonometry**: Students will build the concept on patterns to better understand the Binomial Theorem as they analyze two or more variables which will represent the quantities between two relationship modeled on the coordinate axes with labels. **Chemistry/Biology/Physics:** Students will build equations in two or more variables to represent the relationship of the quantities using graphs and equation to find trend lines and behaviors similar to what students on chemistry/ biology/physics labs using excel.

			● Count the number of elements in a subset by counting the complement of the desired subset ● Use appropriate counting tools and formulas ● Relate different counting strategies to each other ● Use efficient strategies for counting ● Identify isomorphic problems ● apply counting strategies to functions defined on finite sets		

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 5 "Functions and Tables" (5) Weeks

Essential Questions	● **What is the Fundamental Law of Exponents? What are some of its corollaries?** ● **How do you extend the laws of exponents to define zero, negative, and rational exponents?** ● **For $f(x) = b^x$, why is it true that $f(m) \bullet f(n) = f(m+n)$?** ● **Why must an exponential function have an inverse function?** ● **What are some reason to use logarithms?** ● **What is a logarithmic scale and when do you use it?**
Learning Objectives for Unit	Students will learn to ● (extension) by making strategic choices that preserve the laws of exponents to extend the set of numbers that can be used as exponents to include all real numbers ● (visualization) use the graphs of exponential and logarithmic functions to conjecture critical properties of these functions ● (logical reasoning) to solve problems by reasoning about exponential and logarithmic functions using the laws of exponents and the laws of logarithms
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **What are the simplified forms of the expressions 4^0, 7^{-2}, and $5^{27/3}$?** ● **If you invest $1000 in an account at 6% interest, compounded annually, how much will you have after 30 years?** ● **If you invest $1000 in an account at 6% interest, compounded annually, how many years will it take until your money grows to $10000?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **Develop the laws of exponents by extending from positive integers to include zero, negative, and rational numbers** ● **Use sequences to explore exponents** ● **Graph exponential functions and write rules for exponential functions given a table of inputs and outputs or two points on the graph of the function** ● **Evaluate logarithms of any base, to use logarithms in solving exponential equations and to graph logarithmic functions**

		Students are responsible for a unit project that relies on using the learned skills to explore the functional equation that exponential functions satisfy: f(x + y) = f(x) • f(y)to build an understanding of how any function that satisfies this equation must behave. **Assessments: Tests, quizzes, homework, classwork**			
CC Standards/ Lawrence Standards	**Language Objectives**	**Academic Language**	**Content Objectives**	**Texts and Supplemental Learnings**	**Cross-Content Connections**
A.REI.4 Solve quadratic equations in one variable **A.REI.4.b** Solve quadratic equations by inspection taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write the as a+/- bi for real numbers a and b **A.REI.6** Solve systems of linear equations exactly and approximately focusing on pairs of linear equations in two variables **A.SSE.2** Use the structure of an expression to identify was to rewrite it **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by an expression **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems **F.BF.1.a**	● evaluate expressions involving exponents, including zeros, negative, and rational exponents ● summarize how to find missing terms in a geometric sequence and generate geometric sequences to interpret expressions involving rational exponents ● model how to convert between exponential and radical forms for rational exponents ● discuss how to extend the laws of exponents to allow the evaluation of zero, negative, and rational exponents. ● reason logically and state how to verify that a particular interpretation of an exponent flows the laws of exponents. ● generalize from specific examples to develop and verify identities ● evaluate logarithm of any base using a calculator ● model the use logarithms to solve for exponential equations ● graph logarithmic functions ● discuss how to reason logically from the definition of a logarithmic function and the laws of exponents to develop the laws of logarithms ● visualize and model the graph of a logarithmic function from the graph of the corresponding exponential function ● explain how to convert flexibly and strategically between logarithmic form and exponential form, and choose the best from to solve problems	● arithmetic sequence ● b^n ● base ● change-of-base rule ● closed-form definition ● common logarithm, log x ● exponent ● exponential function ● exponential growth ● extension by continuity ● functional equation ● geometric sequence ● inverse function ● laws of exponents ● laws of logarithms ● linear scale ● log-log graph paper ● logarithmic scale ● monotonic ● negative exponent ● one-to-one ● product model ● rational exponent ● real *n*th root ● recursive definition ● semilog graph paper ● strictly decreasing ● strictly increasing ● unit fraction exponent ● zero exponent	*Learning Goals* SWBAT: ● Review the laws of exponents ● Evaluate expressions involving exponents, including zero, negative, and rational exponents ● Find missing terms in a geometric sequence and generate sequences to interpret expressions involving rational exponents ● Convert between exponential and radical forms for rational exponents ● Graph an exponential function to determine the equation of an exponential function given two points on its graph ● Identify an exponential function from the table it generates and use the table to create a closed-form or recursive definition of the function ● Extend the laws of exponents to include all real-numbers exponents and express the laws as function equations ● Evaluate the inverse of the function $y=b^x$ either exactly or by approximation ● Evaluate logarithms of any base using a calculator ● Use logarithms to solve exponential equations ● Graph logarithmic functions ● Graph functions using a logarithmic scale *Habits and Skills* SWBAT: ● Extend the laws of exponents to allow evaluation of zeros, negative and rational exponents ● Reason logically to verify that a particular interpretation of an exponent follows the laws of exponents	**CME Project: Precalculus Common Core-Chapter 5 (pages 340-437)** **https://www.pearsonsuccessnet.com/** **http://cmeproject.edc.org/** **Edmodo** **SMART Labs** **Khan Academy** **YouTube**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Physics:** Students will learn to evaluate functions to reason laws of logarithmic functions and exponents. **Biology:** Logarithmic functions are used to better understand the decay and growth function in terms of cells or species. **Algebra II:** Students will use the basic knowledge learned about logarithmic function to build concepts. **Trigonometry**: Students will expand the knowledge of logarithmic functions by building the concept of exponents and how build geometric series and functions.

Determine an explicit expression, a recursive process, or steps for calculations from a context **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms **F.IF.3** Recognize that sequences are functions sometimes defined recursively, whose domain is a subset of the integers **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function			● Generalize form specific examples to develop and verify identities ● Reason by continuity to extend the definition of exponent to include all real numbers ● Visualize exponential growth by examining graphs and tables of exponential functions ● Draw logical conclusions from the laws of exponents and properties of exponential functions to solve problems and probe conjectures ● Reason logically from the definition of a logarithmic function and from the laws of exponents to develop the laws of logarithms ● Visualize the graph of a logarithmic function from the graph of the corresponding exponential function ● Convert flexibility and strategically between logarithmic form and exponential form, and choose the best form to solve problems		

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 6 "Analytic Geometry" (4) Weeks

Essential Questions	● **What are the basic graphs and their equations (parent functions)?** ● **What are the effects on both the graphs and their equations when the graphs are translated, stretched, shrunk, or reflected?** ● **Why do dilations commute under function composition?** ● **How do you compute the inverse of an affine translation?** ● **How do you use the computed inverse of affine transformation to graph the original equation?** ● **How do describe the effect of an affine transformation on an axis and the effect of changes in the axes on the graph of the equation?** ● **How you identify the fixed points of an affine transformation and the set of affine transformations that fix a particular point?**
Learning Objectives for Unit	Students will learn to

	● (visualization) visualize graphs as transformations of the basic graphs, seeing them as translations, scalings, and reflections or as compositions of these transformations ● (making connections) experiment with changes to an equation and connect the resulting changes in the graph to the type of change in the equation ● (logical reasoning) ask questions about the algebraic structure when faced with new kinds of functions—translations, dilations, and affine translations—to develop conjectures to answer those questions and use logical reasoning to prove their conjectures
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **How are the graphs of $y = x^2$ and $y = (x - 3)^2$ related?** ● **What does the graph of $(x + 1)^2 + (y - 3)^2 = 36$ look like?** ● **What does the graph of $-2y = x^3 - x$ look like?** ● **How do you transform an equation of the form $y = ax^3 + bx^2 + cx + d$ into one of the equations $y = x^3$, $y = x^3 + x$, or $y = x^3 - x$ by composing dilations and translations?** ● **How do you use the replacing-the-axes method to sketch the graph of $y = x^3 + 3x^2 - x + 4$?** ● **How do you use the replacing-the-axes method to explain the relationship between the graphs of $y = x^2$ and $y= (x - 3)^2$?** ● **What is the fixed point for $A_{(5,3)}$?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **Relate the effects on a basic graph and on the equation of the graph of a translation, a scaling, or a reflection of the graph** ● **Write any composition of translations and dilations as an affine transformation and find the inverse of a dilation, a translation, or an affine transformation** ● **Transform an equation into one the basic graphs, describe the effect of an affine transformation on an axis, and identify the fixed appoint on an affine transformation** **Students are responsible for a unit project that relies on using the learned skills to identify the properties that define a group through it resembles abstract algebra. Students will receive practice with function composition, finding inverses for functions, and understanding mathematical definitions and notations.** **Assessments: Tests, quizzes, homework, classwork**

CC Standards/ Lawrence Standards	**Language Objectives**	**Academic Language**	**Content Objectives**	**Texts and Supplemental Learnings**	**Cross-Content Connections**
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise **A.REI.4** Solve quadratic equations in one variable **A.REI.10** Understand that the graph of an equation in two variables is the set of all of solutions plotted in the coordinate plane, often forming a curve (which could be a straight line). **A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x) find the	● sketch basic graphs ● describe the effect of translation of one of the basics graphs on both the graph and the equation of the graph ● describe and model the effect of scaling an axis or reflection on both the graph and the equation of the graph ● compose transformations and sketch/model the effect of such a composition ● visualize and explain variations of the basic graphs under translations, reflections, scaling, and compositions of those transformations ● explore and identify transformations to a graph to a corresponding transformation of its equation	● affine recursive function ● affine transformation ● completing the square ● dilation ● even function ● fixed point ● identity transformation ● intercept ● iterations ● odd function ● reflection transformation ● replacing-the-axes method ● socks and shoes method ● stabilizer	*Learning Goals* SWBAT: ● Sketch basic graphs ● Describe the effect of a translation of one of the basic graphs on both the graph and the equation of the graph ● Describe the effect of scaling an axis or reflections of one of the basic graphs on both the graph and the equation of the graph ● Compose transformation and sketch the effect of such a composition on one of the basic graphs ● Write any composition of translations and dilations as an affine transformation and write any affine transformation as a composition of a dilation and a translation	**CME Project: Precalculus Common Core-Chapter 6 (pages 438-535)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **YouTube** **Glencoe Common Core Precalculus**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Physics:** Students will learn the importance of scaling this is a transferable skill to zoom in and out of aspect of functions or concepts. Students will learn how see one aspect in a bigger or smaller picture depending on the analyses of the concept. **Biology:** Students will learn how to predict transformations and behaviors of function based on what they have

solutions approximately using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/org(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by an expression **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines **A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines **F.BF.3** Identify the effect on the graph of replacing *f(x)=* by *f(x)+k, k f(x), f(kx),* and *f(x +k)* for specific values of k (both positive and negative values); find the value of *k* given the graphs. Experiment with the cases and illustrate an explanation of the effects on the graph using technology **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **G.GMD.4** Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects	● analyze the operation of function compositions on transformations ● write any composition of translations and dilations as an affine transformation and write any affine transformation as a composition of a dilation and a translation ● explain how to find the inverse of a dilation, a translation, or an affine transformation and use that inverse as a tool to solve equations ● model and describe how to find transformations for converting a general quadratic or cubic into one of the basic graph forms ● generalize from a series of numerical calculations to produce a proof for a property of transformations ● summarize how to extend the idea of finding an inverse of a function by undoing its steps in reverse order to find the inverse of an affine transformation ● create connections between algebraic calculations, such as using substitutions to make an equation monic or to complete the square and the affine transformations of translations and dilation ● describe the effect of an affine transformation on an axis and the effect of changes in axes on the graph of an equation ● identify the fixed points of an affine transformation and the set of affine transformations that fix a particular point ● model transformations both as operations on graphs and as operations on coordinate axes ● discuss how to think algebraically to further examine the structure of the set of affine transformations ● summarize how to make connections between transformations on graphs and on coordinate axes	● translation ● unit circle	● Find the inverse of a dilation, a translation, or an affine transformation and use that inverse as a tool to solve equations ● Find transformations for converting a general quadratic or cubic into one of the basic graph forms ● Use the techniques learned to transform an equation into one of the basic graph forms and use the information to graph the original equation ● Describe the effect of an affine transformation on an axis and the effect of changes in axes on the graph of an equation ● Identify the fixed points of an affine transformation and the set of affine transformations that fix a particular point *Habits and Skills* SWBAT: ● Visualize variations of the basic graphs under translations, reflections, scaling, and compositions of these transformations ● Match a transformation of a graph to a corresponding transformation of its equation ● Analyze the operation of function composition on transformations ● Generalize from a series of numerical calculations to produce a proof for a property of transformations ● Extend the idea of finding an inverse of a function by undoing its steps in reverse order to find the inverse of an affine transformation ● Find connections between algebraic calculations, such as using substitutions to make an equation monic or to complete the square, and the affine transformation of translation and dilation		observed to analyzing how inverses work. This is important to biology in the way that students view concepts like genetic mutation or inheritance. **Algebra II:** Students will learn to think about the quadratic function and how the roots relate to functions depending on if the function is even or odd. **Geometry:** Students will start to think of how to use functions and their behaviors to relate to values like roots and exponential functions that alight to said roots. **Trigonometry**: Students will start to think about odd and even function to think about how functions have roots that are solutions and how the solutions affect the behavior of the functions.

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation **G.GPE.2** Derive the equation of a parabola given a focus and directrix **G.GPE.3** Derive the equation of ellipses and hyperbolas given foci and directives **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically **G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling **N.VM.1** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes **N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point **N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors **N.VM.4** Add and subtract vectors **N.VM.4.a** Add vectors end to end components-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitude **N.VM.4.c**			● Visualize transformations both as operations on graphs and as operations on coordinate axes ● Make connections between transformations on graphs and on coordinate axes ● Think algebraically to further examine the structure of the set of affine transformations		

Understand vector subtraction v-w as v +(-w), where -w is the additive inverse of w, with the same magnitude as w and point in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise **N.VM.5** Multiply a vector by a scalar **N.VM.5.a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise **N.VM.5.b** Compute the magnitude of a scalar multiplication cv using ||cv||= |c| ||v|| Compute the direction of cv knowing that when |C| cannot equal 0. the direction of cv is either along v or against v.					

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 7 "Probability and Statistics" (5) Weeks

Essential Questions	• **How do you make a sum table for a function and write a closed-form rule for the sum column where appropriate?** • **How do you describe and use Gauss's method to find the sum of a sequence with a constant difference between successive terms?** • **How do you use Euclid's method to find the sum of a sequence with a constant ratio between successive terms?** • **How do you expand ∑ notation or convert an expanded sum back to ∑ notation?** • **How do you find closed-form expressions for indefinite sums and use them to evaluate definite sums?** • **Can you develop a list of ∑ identities and recognize situations in which you can apply them? How?** • **Can you find closed-form expression for the series associated with a function? How?** • **How you find a closed-form representation for an arithmetic or geometric sequence and it associated series?** • **How do you determine whether a geometric sequence has a limit, and if it does, how do you find the limit?** • **How can you convert a repeating decimal into an exact fraction?** • **How do you generate Pascal's Triangle and evaluate the *n*th row, *k*th column entry as** $\left(\frac{n}{k}\right)$? • **How you notice and explain patterns in Pascal's Triangle?** • **How do you use the Binomial Theorem for expanding expression of the form** $(a+b)^n$**?**

Learning Objectives for Unit	Students will learn to ● (visualize sums) use pictures of arrays of squares and dots to make sense of the closed forms for various sums ● (reason about linearity) verify that the ∑ operation meet the functional equation requirements of linearity— ∑(f+g)=∑f+∑g and ∑(kf)=k(∑f) ● (passing to the limit) develop intuition about the process of finding a limit of a function as one variable get arbitrarily large through experiment and reasoning
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **Describe Gauss's method for summing all integers from 1 to n.** ● **Find a formula for** $\sum_{j=0}^{n} 2^j$ **in terms of *n*.** ● **Evaluate** $\sum_{j=1}^{5} 4j$. ● **What is** $\sum_{k=0}^{25} (k+6^k)$**?** ● **What is a recursive rule for the series associated with f(n) = 3n + 6 having initial term f(0)?** ● **What is a closed form for the following recursive rule?** $f(n)\{5, if\ n=0\ \ f(n-1)+2n^2+3n+2, if\ n>0$ ● **What is an arithmetic sequence?** ● **What is a geometric sequence?** ● **How do you write the repeating decimal 0.121212121212… as a fraction?** ● **What is the sum of the entries in row 10 of Pascal's Triangle?** ● **What is the expanded form of** $(2d+7)^8$**?** ● **What is the coefficient of** $x^7 y^3$ **in the expansion of** $(x+y)^{10}$**?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **Make a sum table for a function, write closed-form rules for the sum column, and use ∑ notation** ● **Use Gauss's method to find the sum of a sequence with constant differences and Euclid's method to find the sum of a sequence with constant ratios** ● **Find closed-form definitions for indefinite sums and to use the definitions to evaluate definite sums, develop and use ∑ identities, and find closed-form expression for the series associated with a function** ● **Find closed-form for arithmetic and geometric sequences and their associated series** ● **Generate Pascal's Triangle, to find and explain patterns in Pascal's Triangle, and to use the Binomial Theorem to expand expressions** **Students are responsible for a unit project that relies on using the learned skills to use the line of best fit to derive the actual formula that their calculators use to find the line of best fit for a data set.** **Assessments: Tests, quizzes, homework, classwork**

CC Standards/ Lawrence Standards	Language Objectives	Academic Language	Content Objectives	Texts and Supplemental Learnings	Cross-Content Connections
A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle **F.BF.1.a** Determine an explicit expression, a recursive process, or steps for calculations from a context **N.Q.1**	● reason logically to understand how both Gauss's method and Euclid's methods work and choose which method to used for finding a sum and explain why, both verbally and in writing ● visualize and model a sum geometrically to make sense of an algebraic pattern ● generalize and explain a result from a series of numerical examples	● arithmetic sequence ● arithmetic series ● associated series ● Bernoulli's formulas ● Binomial Theorem ● common difference ● common ratio ● constant differences ● definite sum ● Euclid's method ● figurate numbers ● Gauss's method ● geometric sequence ● geometric series ● identity	*Learning Goals* SWBAT: ● Make a sum table for a function and write a closed-form rule for the sum column where appropriate ● Use Gauss's method to find the sum of a sequence with a constant difference between successive terms ● Use Euclid's method to find the sum of a sequence with a constant ration between success terms ● Expand ∑ notation or convert an expanded sum back to ∑ notation ● Find closed-form expression for indefinite sums and sued them to evaluate definite sums ● Find closed-form expression for the series associated with a function	**CME Project: Precalculus Common Core-Chapter 7 (pages 536-663)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **YouTube**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Physics:** This uit considers the probability and accuracy of data introducing concepts of how

Use units as a way to understand problems, and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data display **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities **S.CP.1** Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events **S.CP.2** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent **S.CP.3** Understand that conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities **S.CP.5** Recognize and explain the concepts of conditional probability and independence on everyday language and everyday situations **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and	● develop a list of ∑ identities and recognize situation in which to apply them ● find and model closed-form expression for the series associated with a function ● generalize the steps in calculating definite sums to find a closed form for the indefinite sum ● reason logically using a recursive definition of a function to develop a closed-form definition ● distinguishes between a geometric and a arithmetic sequence and find its closed-form ● determine whether a geometric sequence has a limit, and if it does, how to find the limit ● reason logically to understand, write, and analyze proofs ● generate Pascal's Triangle and evaluate the nth row, kth column entry as $\left(\frac{n}{k}\right)$ ● notice and explain patterns in Pascal's Triangle ● seek invariants or regularity in calculation to develop a conjecture ● reason logically to prove conjectures ●	● indefinite sum ● index ● limit ● Pascal's Triangle ● repeating decimal ● series associated with f ● term ● sigma ● summation	● Find a closed-form representation for an arithmetic sequence and its associated series ● Find a closed-form representation for a geometric sequence and its associated series ● Convert a repeating decimal into an exact fraction ● Use the Binomial Theorem for expanding expressions of the form $(a+b)^n$ *Habits and Skills* SWBAT: ● Reason logically to understand how both Gauss's method and Euclid's method work, and choose which method to use for finding a particular sum ● Visualize a sum geometrically to make sense of an algebraic pattern ● Generalize a result from a series of numerical examples ● Visualize a complicated sum as a combination of different simpler sums ● Generalize the steps in calculating definite sums to find a closed form for an indefinite sum ● Visualize arithmetic and geometric series to better understand their behavior ● Think about extreme cases as values of n become very large or terms of a sequence become very small	**Glencoe Common Core Precalculus**	data is used in a distribution curve **Biology:** Students will think about the concepts of sampling and how samples can be used to study the chance of something happening or the likelihood of something like a virus can affect a population **Algebra II:** Students will apply the concepts of probability to think about the independence of trails and how likely the event is to happen given certain situations **Geometry:** Students will make sense of geometric patterns that will apply to make algebraic ones so that students can model summations and build an understanding about value **Trigonometry**: Students will think logically about the probability and statistics as they see the possibility of an outcome using concepts learned to draw conclusions **Chemistry:** Students will learn about sample size and how data is relevant. Students will learn about the normal distribution curve and what the expectations and variations might be. **History:** Students will learn how to read information as factual and how the data can be interpreted to see the validity given the population/sample size. **English/Speech:** Students will learn how to validate the information given the sample size and the spread

interpret the answers in terms of the model **S.CP.7** Apply the Additional Rule P(a or B)= P(A)+P(B)-P(A or B), and interpret the answers in terms of the model **S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B)=P(A) P(A|B)=P(B)P(A|B), and interpret the answer in terms of the model **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems **S.IC.2** Decide if a specified model is consistent with results from a given data-generating process using simulation **S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling **S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant **S.IC.6** Evaluate reports based on data **S.ID.1** Represent data with plots on the real number line **S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers) **S.MD.1** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same					of the confidence interval by applying validity to the information given and understanding how probability works give the distribution

graphical displays as for data distributions **S.MD.2** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities are assigned empirically; find the expected value **S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; finding the expected value **S.MD.5** Weigh the possibilities outcomes of a decision by assigning probabilities to payoff values and finding expected values **S.MD.5.a** Find the expected payoff for a game of chance **S.MD.5.b** Evaluate and compare strategies on the basis of expected values **S.MD.6** Use probabilities to make fair decisions **S.MD.7** Analyze decisions and strategies using probability concepts					

Abbott Lawrence Academy 2018-2019 Curriculum Map:
Subject: Advanced Honors Precalculus Grade: 11th
Unit 8 "Ideas of Calculus" (6) Weeks

Essential Questions	● **How you find the area under an irregular shape?** ● **What is the area under a curve? How do can I find said area exactly?** ● **What is Fermat's approach to finding the area under the graph given two points?** ● **How can areas under a curve be used to gram new perspectives on familiar functions?** ● **How is area related to logarithmic functions?**
Learning Objectives for Unit	Students will learn to ● (approximation) apply given information to approximate values leading that approximations can lead to exact values.

	● (passing to the limit) view the limit of a sequence by getting as close to the value as possible. ● (reasoning continuity) learn about the area of the curve and how it is reasonable to suppose that the function is continuous and small changes will also produce changes in the area of said curve
Performance tasks: Formative and Summative	**SWBAT answer the following types of questions:** ● **How can you find the area of an irregularly-shaped figure?** ● **How can you estimate the area under a curve?** ● **What is the area under a curve given a boundary?** **Students will have to complete Mathematical Reflections for the various investigations that have students think and write about how to:** ● **approximate the areas of irregular shapes and regions under curves by using squares and rectangles** ● **use Cavalieri extended Archimedes's result to find the area under the graph for integers** ● **follow the historical approach taken by two mathematicians to find the area under certain curves** **Students are responsible for a unit project that relies on using the learned skills to use the sequence of polynomials called the Chebyshev polynomials to produce multiple angle formulas for cosine..** **Assessments: Tests, quizzes, homework, classwork**

CC Standards/ Lawrence Standards	Language Objectives	Academic Language	Content Objectives	Texts and Supplemental Learnings	Cross-Content Connections
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems **F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents	● write a program to compute upper and lower area approximations ● find a closed form expression for an area approximation ●model a recursive function definition ●use the Archimedes' method to approximate an area ●Use Fermat's method to approximate an area ● define the area under the graph and measure the area ●define Cavalieri's method of dividing intervals into rectangles to derive the formula for the sums of power ●explain how to use rectangles to estimate the areas of irregular shapes	● Fermat lower sum ● Fermat upper sum ● lower sum ● upper sum	*Learning Goals* SWBAT: ● Estimate the areas of irregularly-shaped objects ●Estimate the area under the graph of $y=x^2$ between x=0 and x=1 ● Calculate the area under the graph of $y=x^2$ between x=0 and x=1 exactly ● Find the area under the graph of $y=x^3$ between x=0 and x-1 ● Calculate the area under the graph of $y=x^m$ between x=0 and x=1 for any positive integer m ● Develop formulas for calculating $S[1,a](x^m)$ where m is any integer ●investigate a mysteriously familiar function L(a) ●Find the area under the graph of $y=e^x$ between x=0 and x=1 *Habits and Skills* SWBAT: ● Use rectangles to estimate the area of irregular shapes ● Use approximations to find areas to any desired level of accuracy ●Use the formula for the sum of squares to find areas	**CME Project: Precalculus Common Core-Chapter 8(pages 664-737)** https://www.pearsonsuccessnet.com/ http://cmeproject.edc.org/ **Edmodo** **SMART Labs** **Khan Academy** **YouTube** **Glencoe Common Core Precalculus**	**All subjects:** The students will consider the text/lecture and collaborative work to engage/learn with their notes. They will create interactive notebooks based off of readings, lectures, collaborative work, assignments, etc. **Physics:** Finding the area under the curve can reveal key concepts like the velocity an object is traveling **Biology:** The area under the curve can be used to find concepts of how much water is in a pool for a tad pool or if students need to find the volume of a liquid when there is a certain shape **Algebra II:** Students will use the concepts learned in Algebra II to extend the concept of functions

			●Use the closed forms for summations to find areas ● Use a historical perspective to make sense of the most important ideas of calculus ● Use a CAS to make short work of complicated calculations ● Use properties of a mystery function to identify the function ●Use areas under the curves to gain new perspectives on familiar functions		**Geometry:** Students are working with area formulas to use basic concepts to extend and find the area of the shapes and curves **Trigonometry**: Students will continue to expand the concepts of summation by looking at the area under the curve as a series of rectangles that will be added **Chemistry:** Students will learn key concepts to approximate values to better understand how formulas develop a key concept that extends to building formulas or chemical compounds **History:** Students will be learning about key mathematicians that have impacted technology and its uses. Students will learn about the significant of mathematical knowledge and how it is still relevant **English/Speech:** Students will learn to construct arguments and explanation to justify their findings and justify what it is that they are mathematically modeling